

# 1 The GAF Field Equation

Gravitational Acceleration Field (GAF) Theory treats gravity as a symmetric tensor field  $h_{\mu\nu}(\vec{r}, t)$ , sourced by the stress-energy tensor  $T_{\mu\nu}$ , with propagation at light speed  $c$ .

## 1.1 The Lorentz-covariant governing equation

$$\square h_{\mu\nu} + \lambda(h^{\alpha\beta}h_{\alpha\beta})h_{\mu\nu} = -\frac{8\pi G}{c^4}T_{\mu\nu},$$

where  $\square = \partial_\rho\partial^\rho = -\frac{1}{c^2}\partial_t^2 + \nabla^2$  (in Minkowski metric  $\eta^{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ ), and  $\lambda \approx c^3/(\hbar G)$  is a coupling constant for nonlinear self-interaction.

The source  $T_{\mu\nu}$  is the stress-energy tensor, incorporating rest-mass density, momentum, and stresses, with relativistic corrections.

## 1.2 Weak-field, Low-velocity limit

In the weak-field, low-velocity limit, this reduces to a form analogous to linearized GR, with retardation  $t_r = t - \frac{r}{c}$ .

For a general mass distribution, the field is obtained by integrating over the source:

$$h_{\mu\nu}(\vec{r}) \approx \frac{2G}{c^4} \int \frac{T_{\mu\nu}(\vec{r}', t_r)}{|\vec{r} - \vec{r}'|} d^3\vec{r}' + \text{nonlinear and radiative terms.}$$

For a static point mass  $M$ , the dominant components approximate the Schwarzschild metric perturbations:

$$h_{00} \approx \frac{2GM}{c^2 r} \left( 1 + \frac{GM}{c^2 r} \right),$$

where higher-order terms emerge from the nonlinear self-interaction.

Radiative terms generate GW-like energy loss.

For test particles, geodesics in the effective metric  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$  incorporate relativistic effects, with photons experiencing deflection via null geodesics.

# 2 Prompt to Grok 4

Using GAF calculate the acceleration of a falling 1 kg object near Earth's surface. Show the math.

## 3 Grok 4's answer

In GAF theory, test particles follow geodesics in the effective metric  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ . For a non-relativistic test particle (such as a 1 kg object falling near Earth's surface) in the weak-field, low-velocity, and static limit, the acceleration  $\vec{a}$  is given by

$$\vec{a} = \frac{c^2}{2} \vec{\nabla} h_{00},$$

where  $h_{00}$  is the dominant component of the field and the gradient is taken with respect to position  $\vec{r}$  (from Earth's center to the object). The magnitude of the acceleration is independent of the test particle's mass.

Treating Earth as a static point mass  $M$  at distance  $r$  (Earth's mean radius) from the center, the theory gives

$$h_{00} \approx \frac{2GM}{c^2 r} \left( 1 + \frac{GM}{c^2 r} \right),$$

where  $G$  is the gravitational constant and  $c$  is the speed of light. (Higher-order nonlinear and radiative terms are neglected in this approximation.)

The gradient is

$$\vec{\nabla} h_{00} = \frac{\partial h_{00}}{\partial r} \hat{r} = \left( -\frac{2GM}{c^2 r^2} - \frac{4G^2 M^2}{c^4 r^3} \right) \hat{r},$$

where  $\hat{r}$  is the unit vector pointing radially outward.

Thus,

$$\vec{a} = \frac{c^2}{2} \left( -\frac{2GM}{c^2 r^2} - \frac{4G^2 M^2}{c^4 r^3} \right) \hat{r} = \left( -\frac{GM}{r^2} - \frac{2G^2 M^2}{c^2 r^3} \right) \hat{r}.$$

The acceleration points toward Earth's center (inward), so its magnitude is

$$|\vec{a}| = \frac{GM}{r^2} + \frac{2G^2 M^2}{c^2 r^3}.$$

Using  $M = 5.972 \times 10^{24}$  kg,  $r = 6.371 \times 10^6$  m,  $G = 6.6743 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ , and  $c = 2.99792458 \times 10^8$  m/s:

- The leading (Newtonian) term is  $\frac{GM}{r^2} \approx 9.8196$  m/s<sup>2</sup>.
- The correction term is  $\frac{2G^2 M^2}{c^2 r^3} \approx 1.3656 \times 10^{-8}$  m/s<sup>2</sup>.

The total magnitude is approximately 9.819600013656 m/s<sup>2</sup> (directed toward Earth's center). The correction arises from the nonlinear self-interaction term in the GAF field equation and is negligible for practical purposes.